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Student Understanding of Tunneling in Quantum Mechanics

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We are studying student understanding of the phenomenon of quantum tunneling through a potential barrier, a standard topic in most introductory quantum physics courses. Series of interviews and tests revealed that many students believe energy is lost in the tunneling process. A survey was designed to investigate the prevalence of the energy-loss idea. This survey was administered to Physics special and engineering Physics special students of the department of Physics, University of Colombo.

1. INTRODUCTION

Compared with other areas of the physics curriculum, quantum mechanics has received little attention from physics education researchers [1, 2]. We have begun a project investigating student understanding of tunneling in quantum mechanics. Though this is a standard topic taught in most introductory quantum mechanics courses, we are finding that students do not possess well-defined mental models of the tunneling process.

2. STUDENT DESCRIPTIONS OF ENERGY LOSS

The project began with a multiple choice question test and a series of interviews with conventional and engineering Physics special students. They were in the final year of their studies and all have followed an introductory course and a advanced course in quantum mechanics.

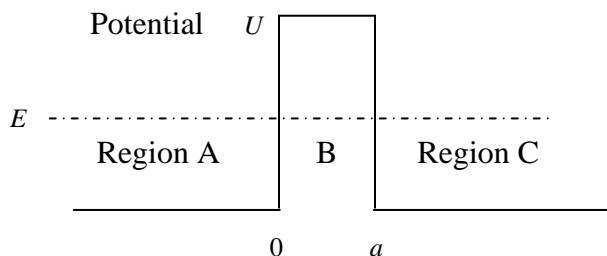


Figure 1: Square Barrier

A remarkable similarity emerged in the responses of most of the students. They articulated that energy was lost during the tunneling process. An excerpt from an interview with a

student reveals a typical student response. The student has been shown a square barrier (see Figure 1) and asked about a stream of particles with kinetic energy half the value of the energy of the barrier.

Question 1: How does the electron's energy in Region C compare to its energy in Region A?

Answer: It's less.

Question 2: Ok, why is it less?

Answer: Because it requires energy to go through this barrier.

Another student proposed the energy loss idea without being specifically questioned about energy.

Question 1: Is there any chance the electron will ever be found in Region C?

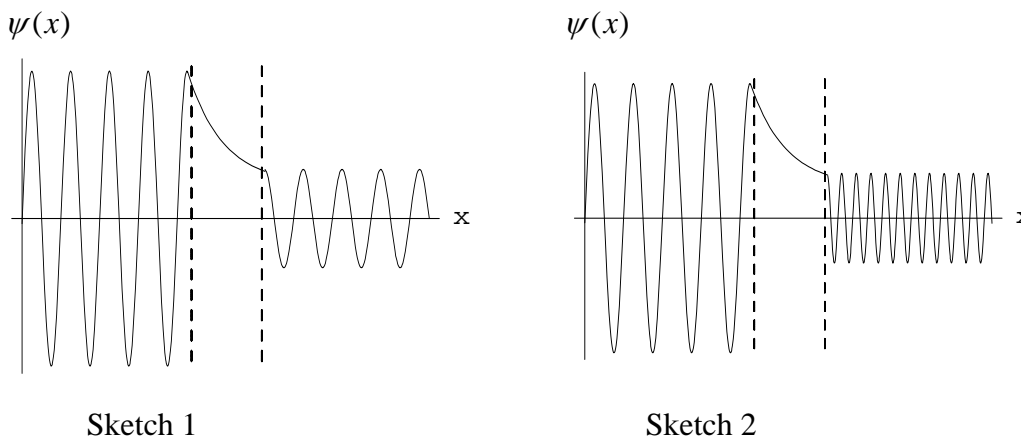
Answer: Yes, there is.

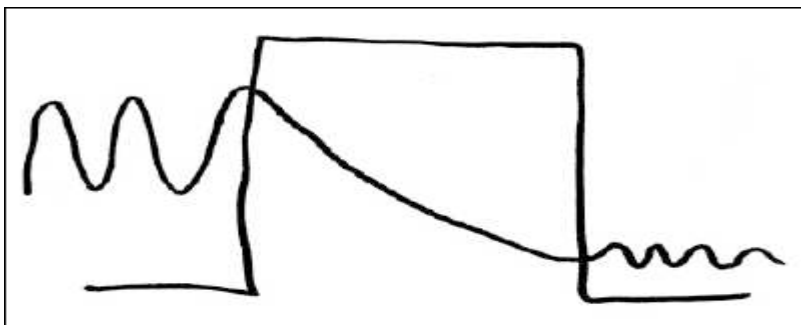
Question 2: There is? How do you know that?

Answer: Well I know because I was taught that when the particle of certain energy encounters a potential barrier, there is a possibility that the particle will just go straight losing energy as it does so, and come out on the other side of the potential barrier at a lower energy and continue on its path.

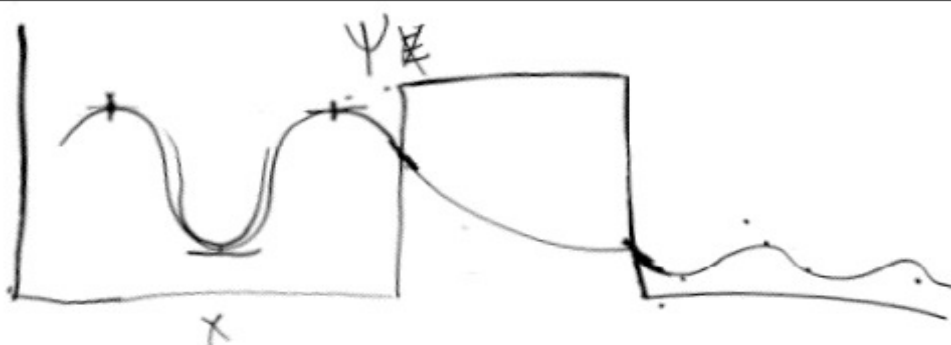
Examining the interview transcripts can provide some insight into students reasoning, Student sketches of the wave function in all three regions are shown in Figure 2.

Figure 2: Student sketches of the wave function.





Sketch 3



Sketch 4

In both sketches, the students have correctly drawn the wave function as sinusoidal in Regions A and C, as well as exponentially decaying in Region B. In addition, the amplitude of the wave function in Region C is smaller than the amplitude in Region A. During interviews, all students discussed the connection between the amplitude of the wave function and the probability of locating the particle. If a student is questioned about the probability of detecting a particle in Region C, examination of the amplitude of the wave function in Regions A and C will yield the correct response. However, students have incorrectly shifted the wave function to a lower average position in Region C. If a student has this picture of the wave function in mind, sketched on an energy-versus-position graph (note the original label of “E” on the vertical axis of the sketch 4), he or she would likely reason that the particle has lost energy. Results indicate that students often use axis height to indicate particle energy in quantum tunneling situations.

The sketches reveal what may be a source of additional confusion for students. Many students have first sketched the square barrier, even though they were merely asked to sketch the wave function as a function of position. The vertical axis for the barrier diagram represents energy, but the vertical axis for the wave function sketch represents the amplitude of the wave function. Instructors often sketch the wave function on top of energy diagrams. Perhaps students, in this instructional environment, incorrectly link the amplitude

of the wave function to the energy. Equipped with this model, it is reasonable that students examining the decaying wave function in Region B would conclude that the particle was losing energy.

3. REFINING OUR UNDERSTANDING OF STUDENT THINKING

In order to investigate the prevalence of the notion that energy is lost during tunneling through a barrier, a survey was designed to ask students about energy loss and probability of detection for particles tunneling through square barriers.

The survey begins by showing students a diagram of a square barrier (similar to Figure 1), and asking about the energy of particles that are detected in Region C. The results of question 1 are shown in Table 1.

Table 1: Responses to Question 1

| <u>Response</u> (correct is in bold) | <u>Number of students</u> (n = 14) |
|--|------------------------------------|
| <i>Energy in Region C is the same as the energy in Region A</i> | 3 (21%) |
| <i>Energy in Region C is less than the energy in Region A</i> | 11 (79%) |

Students were then prompted to explain the reasoning used to determine their response. For example:

- “Some energy is dissipated as the particle tunnels through the potential barrier”
- “It will take some energy for the particles to penetrate the barrier in Region B”
- “Energy is ‘lost’ getting through the barrier”
- “The potential barrier Region B lessens the energy of the particles”
- “Particle should lose energy tunneling through a barrier”

We note that the best students, though they performed better on the survey than the others, as expected, gave what seemed to be memorized or incomplete explanations of their reasoning. Responses included “particles are able to tunnel,” and “the particles don’t lose energy when they tunnel”. These responses suggest that while advanced students have perhaps memorized the correct answers, their understanding of tunneling phenomena may be no better than that of others.

The remainder of the survey dealt with the effect of modifying either the potential barrier (increasing width or height) or the particle energy (decreasing, increasing below the barrier

energy level, or increasing above the barrier energy level). In each scenario, students were asked about (i) the probability of detection, and (ii) the energy of the particles in Region C.

Examining the responses of the students on the remainder of the survey (see Table 2), it is apparent that they perform better on questions about probability than on those about energy. While this may indicate good mental models for probability, it may also suggest that students don't have well-defined connections between the various concepts involved in tunneling. Many [1] have described student difficulties in reasoning about the behavior of waves. A student model based on the wave function portrayed in Figure 2 might suggest that everything – amplitude, energy, probability – decreases after tunneling. On the administered survey, such reasoning would suggest decreased probability (correct), as well as decreased energy (incorrect).

Table 2: Additional Survey Responses – Students ($n = 14$)

| <u>Scenario</u> | Probability Question Answered Correctly | Energy Question Answered Correctly |
|--|---|------------------------------------|
| <i>Barrier Width is Doubled</i> | 14 (100%) | 3 (21%) |
| <i>Barrier Height is Doubled</i> | 13 (93%) | 9 (64%) |
| <i>Particle Energy is Increased (but below barrier energy)</i> | 12 (86%) | 5 (36%) |
| <i>Particle Energy is Decreased</i> | 13 (93%) | 3 (21%) |

Some students seem to think of physical, macroscopic tunnels when they reason about quantum mechanical tunneling. Examining the survey responses from the students seems to support this hypothesis. In the scenario where the *width* of the barrier is increased, the largest percentage (79%) answer that the energy loss is now greater. In the scenario where the *height* of the barrier is increased, the largest percentage (36%) also answer that the energy loss is unaffected by this increase. For us as researchers, this suggests an analogy to macroscopic tunneling; it does take more energy to tunnel through a *wider* mountain, but does not take more energy to tunnel through a *higher* mountain.

4. CONCLUSIONS

Our work in investigating student understanding of tunneling has revealed that students use many ideas both successfully and unsuccessfully. As examples of effective reasoning, we note that most students are comfortable enough with the idea of a wave function to be able to reason about its sinusoidal or exponential nature in domains where the particle's average energy is both greater than and less than the potential energy of the region. Students also apply the ideas of continuity to the wave function, as evidenced in interview responses as well as an examination of their sketches of the wave function in the three regions. Furthermore, students seem to accept the notion of tunneling; no student interviewed or surveyed responded that tunneling was impossible on the microscale.

Students also reasoned incorrectly in many areas. Our most common result was the idea that energy is lost by a tunneling particle. We believe that this is due to several reasons, including:

- misinterpretation of the graphical representations, specifically the vertical axis of the graph of the wave function;
- common sense ideas of objects passing through barriers; and
- an explicit analogy to macroscopic situations, such as building a tunnel through a mountain.

Further research (probably with students in the other universities) might indicate specific difficulties in interpreting elements of the wave function (such as amplitude or wavelength) that are consistent with the literature. [3]

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